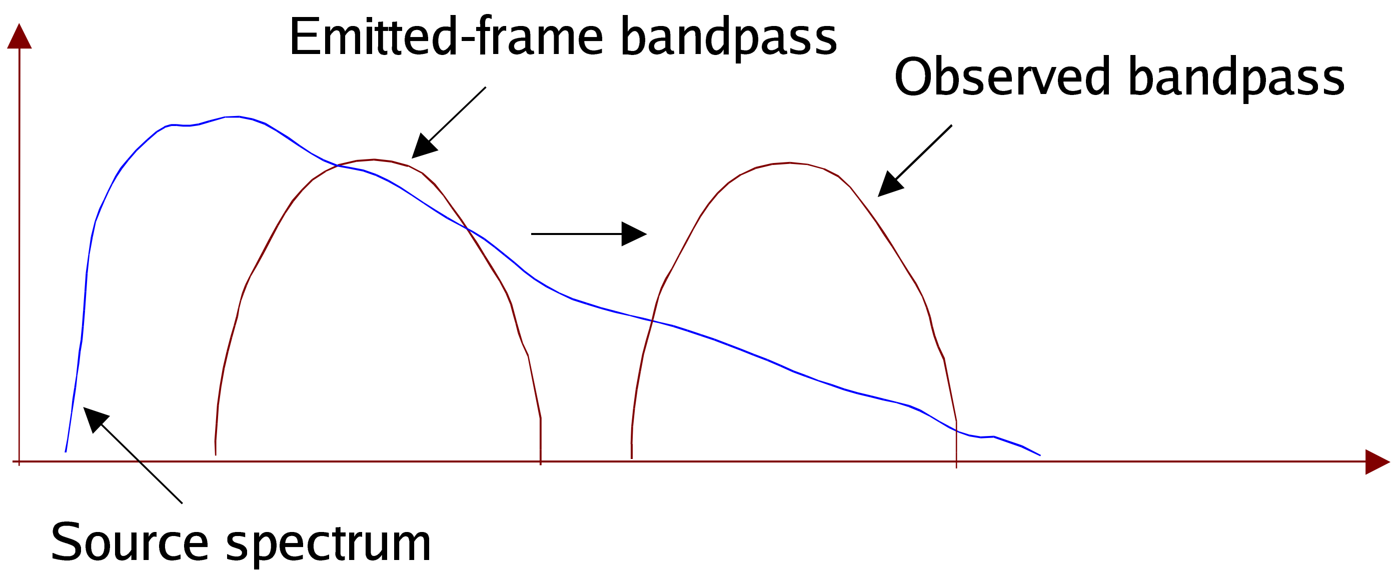
K-Corrections – what are they?

- In a word, K-corrections are used to relate a source spectrum from its emitted-frame bandpass to the observed bandpass.



K-Corrections Definition (1)

- Let's consider a source observed to have apparent magnitude m_R when observed through photometric bandpass R (Alex' y), for which one wishes to know its absolute magnitude M_Q in emitted-frame bandpass Q (Alex' x). Ignoring extinction, the K-correction K_{QR} for the source is defined by:

$$m_R = M_Q + DM + K_{QR} \quad ,$$

  - Where DM is the distance modulus $DM = 5\,\log_{10}\left[\frac{D_L}{10\ \mathrm{pc}}\right] \quad ,$

- Now, the apparent magnitude m_R of the source is related to its spectral flux density $f_\nu(\nu)$ (energy per unit time per unit area per unit frequency) by:

$$m_R = -2.5\,\log_{10}\left[\frac{\int \frac{\mathrm{d}\nu_o}{\nu_o}\, f_\nu(\nu_o)\, R(\nu_o)}{\int \frac{\mathrm{d}\nu_o}{\nu_o}\, g_\nu^R(\nu_o)\, R(\nu_o)}\right] \quad ,$$

  - Where the integrals are over the observed frequences ν_0; $g_\nu^R(\nu)$ is the spectral flux density for the "standard" source – e.g., Vega.

K-Corrections Definition (2)

- The absolute magnitde M_q is defined to be the apparent magnitude that the source would have if it were 10 pc away at rest (not redshifted!). It's realted to the spectral luminosity density $L_\nu(\nu)$ (energy per unit time per unit frequency) of the source by:

$$M_Q = -2.5 \log_{10}\left[\frac{\int \frac{\mathrm{d}\nu_e}{\nu_e}\,\frac{L_\nu(\nu_e)}{4\pi\,(10\ \mathrm{pc})^2}\,Q(\nu_e)}{\int \frac{\mathrm{d}\nu_e}{\nu_e}\,g_\nu^Q(\nu_e)\,Q(\nu_e)}\right] \quad ,$$

  - where the integrals are over the emitted (rest-frame) frequences ν_e

- If the source is at redshift z, its luminosity L is related to its flux by:

$$L_\nu(\nu_e) = \frac{4\pi\,D_L^2}{1+z}\,f_\nu(\nu_o) \quad ,$$

$$\nu_e = [1+z]\,\nu_o \quad .$$

K-Corrections Definition (3)

- Putting everything together, we see that

$$K_{QR} = -2.5 \log_{10} \left[[1+z] \frac{\int \frac{\mathrm{d}\nu_o}{\nu_o} f_\nu(\nu_o) R(\nu_o) \int \frac{\mathrm{d}\nu_e}{\nu_e} g_\nu^Q(\nu_e) Q(\nu_e)}{\int \frac{\mathrm{d}\nu_o}{\nu_o} g_\nu^R(\nu_o) R(\nu_o) \int \frac{\mathrm{d}\nu_e}{\nu_e} f_\nu\left(\frac{\nu_e}{1+z}\right) Q(\nu_e)} \right] \quad .$$

- If we use wavelength instead of frequency, we get:

$$K_{QR} = -2.5 \log_{10} \left[\frac{1}{[1+z]} \frac{\int \mathrm{d}\lambda_o \, \lambda_o \, f_\lambda(\lambda_o) R(\lambda_o) \int \mathrm{d}\lambda_e \, \lambda_e \, g_\lambda^Q(\lambda_e) Q(\lambda_e)}{\int \mathrm{d}\lambda_o \, \lambda_o \, g_\lambda^R(\lambda_o) R(\lambda_o) \int \mathrm{d}\lambda_e \, \lambda_e \, f_\lambda([1+z]\lambda_e) Q(\lambda_e)} \right] \quad ,$$

Reconciling with the paper...

- The last equation can be reduced to eqn. (4) in the paper:

$$K_{xy}^{\mathrm{counts}} = -2.5\log\left(\frac{\int \lambda Z(\lambda)S_x(\lambda)d\lambda}{\int \lambda Z(\lambda)S_y(\lambda)d\lambda}\right) + 2.5\log(1+z)$$
$$+2.5\log\left(\frac{\int \lambda F(\lambda)S_x(\lambda)d\lambda}{\int \lambda F(\lambda/(1+z))S_y(\lambda))d\lambda}\right).$$

  - There appears to be a distinction made between "classical" k-corrections and ones computed for photon counting devices that becomes important for precision cosmological measurements (see Appendix).

- The more general version of eqn $m_R = M_Q + DM + K_{QR}$, is

$$m_y(t(1+z)) = M_x(t,s) + K_{xy}(z,t,s,A_x,A_x)$$
$$+\mu(z,\Omega_M,\Omega_\Lambda,H_o) + A_x(t) + A_y(t).$$

- K-corrections depend on everything that affects the observed spectrum – redshift *z*, epoch *t*, stretch *s*, host galaxy and Milky Way extinctions *A*.

The recipe for determining K-corrections

- There is a clear correlation between K-corrections and the B-V color of the SN (Fig.3)

- Given that, the recipe for determining K-corrections is
  - Create a tabulated SN Ia spectrum, and based on that, come up with a grid of magnitudes in standard filters
  - Adjust the colors: account for extinction and the color differences as a function of the measured strech factor
  - Then calculate the K-corrections using eqn. on the previous slide, using integrals over the color-adjusted grid
  - Propagate uncertainties

Interstellar Extinction: Glossary

- **Total interstellar extinction** is characterized by $A(\lambda)$, the extinction in magnitudes as a function of wavelength λ.
    - A depends both on the distance to the object (it is larger for more distant objects on the same line of sight), and on the direction to the object (the interstellar medium is not uniform).
- Not only is the intensity of the light decreased (extinction), it also gets reddened, because the blue component of light is more easily scattered than the red component.
- **Selective extinction** or **reddening** E(B-V) is defined as

  $(B-V)_{obs} - (B-V)_0$, where the subscripts refer to the observed and intrinsic colors of the target.
- Extinction and reddening are related.

Extinction: CCM model

- Let $A(\lambda)$ = the absolute extinction at wavelength λ.
- It is usually expressed relative to the absolute exinction at some reference (typically visible) wavelength, $A(\lambda_{ref}) = A(V)$
- In 1989, Cardelli, Clayton, and Mathis (CCM) found the following expression for a mean interstellar extinction:

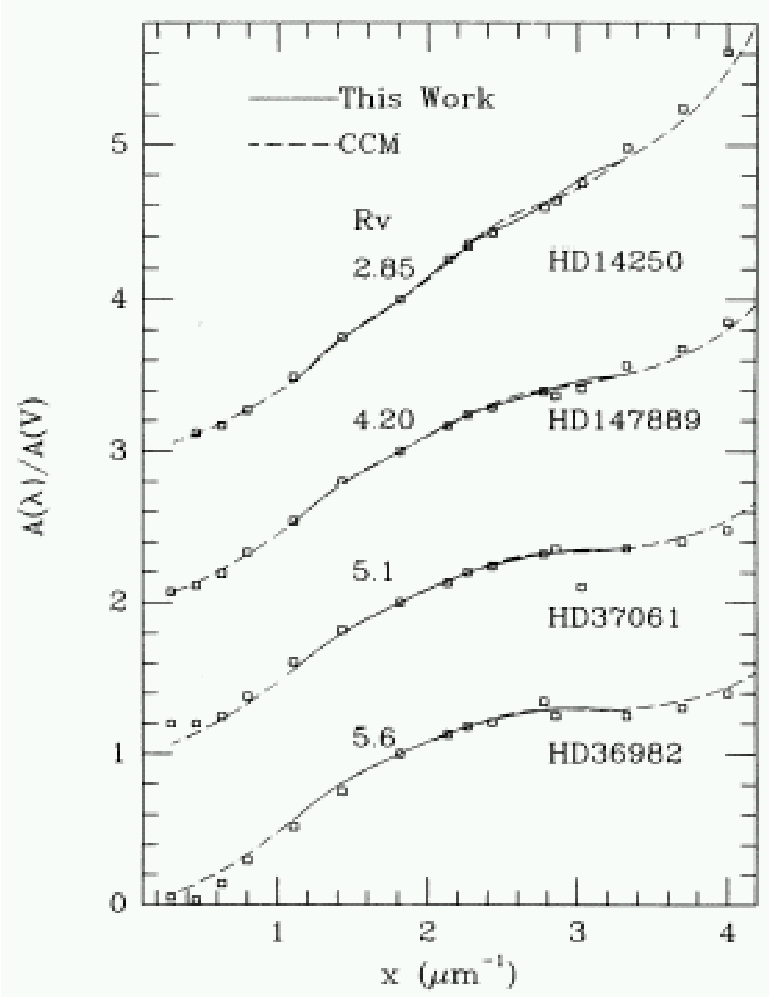
$$A(x)/A(V) = a(x) + b(x)/R_v$$

  - where R_v is the ratio of total to selective extinction $A(V)/E(B-V)$, and $x = 1/\lambda$
  - The average mean value for the diffuse interstellar medium R_v is typically taken to be 3.1

O'Donnell's additions

- O'Donnell's paper extends the CCM law by measurements made in the Stromgen *uvby* filter system

- In general, he finds good agreement with the CCM law, although at small values of R_v his extinction law deviates somewhat from CCM.



SN Ia decline rate and reddening

- It's important to distinguish between the “true” color exceess, $E(B\text{-}V)_{true}$, which is a measure if the absorbing gas and dust along the line of sight; and an observed extinction $E(B\text{-}V)_{obs}$, which varies with the supernova epoch and total extinction.
- There exists the following relationship between the true decline rate and the observed decline rate as a function of $E(B\text{-}V)_{true}$:

$$\Delta \mathrm{m}_{15}(B)_{true} \simeq \Delta \mathrm{m}_{15}(B)_{obs} + 0.1E(B-V)_{true}$$

  - So that from peak to +15 days after the peak the lightcurves get broader.